

ON CONSERVATIVE SPATIAL DISCRETIZATIONS FOR QUASI-GASDYNAMIC SYSTEMS OF EQUATIONS

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ABSTRACT

Quasi-gasdynamic (QGD) and quasi-hydrodynamic (QHD) systems of equations underlie the design of a class of finite-difference methods for solving gas- and fluid dynamic problems [1]. They are specific parabolic regularizations of the compressible Navier–Stokes systems of equations.

A multidimensional barotropic QGD system of equations in the form of mass and momentum conservation laws with a general gas equation of state $p = p(\rho)$ with $p'(\rho) > 0$ and a potential body force is considered [2]. For this system, two new symmetric spatial discretizations on nonuniform rectangular grids are constructed (in which the density ρ and velocity \mathbf{u} are defined on the basic grid, while the components of the regularized mass flux and the viscous stress tensor are defined on staggered grids). These discretizations involve nonstandard approximations for $\nabla p(\rho)$, $\text{div}(\rho\mathbf{u})$ and ρ . As a result, a discrete total mass conservation law and energy inequality guaranteeing that the total energy does not grow with time can be derived. Importantly, these discretizations have the additional property of being well-balanced for equilibrium solutions [3]. For the simpler barotropic QHD system, the related simplifications of the constructed discretizations keep similar properties.

Importantly, the study is extended to the full (non-barotropic) multidimensional QGD system of equations, and the specific spatial discretization having the property of the non-decreasing total entropy is designed, cp. [4].

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