# Application of Consistent and Anti-consistent Functions to Searching for Global Extrema of Functions of Exponential Takagi Class

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A full summary of the results presented here can be found in [1]. These results were obtained independently of the results of the work [2] and the works it refers to. Everywhere continuous, but nowhere differentiable Takagi function on $\mathbb{R}$ was first described in the article [3].

**Definition 1.** *Power series* $c_0 + c_1x + c_2x^2 + \ldots$ is called *unitary* if the free term $c_0$ equals 1, and other coefficients $c_n$ $(n = 1, 2, 3, \ldots)$ equal either $-1$ or 1.

**Definition 2.** Suppose that $w \in \mathbb{R}$ and $F(x) = c_0 + c_1x + c_2x^2 + \ldots$ is a unitary series. Then $F(x)$ is called *series consistent with the point* $w$, and $w$ is called *the point consistent with the series* $F(x)$, if for each $k = 1, 2, 3, \ldots$ the following inequality holds:

$$c_k \cdot (c_0 + c_1w + \ldots + c_{k-1}w^{k-1}) < 0.$$

**Theorem.** Пусть $v \in (-1; 1)$ и с точкой $2v$ согласован унитарный ряд $F_{2v}(x) = c_0 + c_1x + \ldots + c_nx^n + \ldots$. Тогда верны следующие утверждения:

Let $v \in (-1; 1)$ and the unitary series $F_{2v}(x) = c_0 + c_1x + \ldots + c_nx^n + \ldots$ be consistent with the point $2v$. Then the following statements are true:

1) the set of points of the global maximum of the function $T_v$ on the segment $[0; 1]$ contains only two (possibly coinciding) points: the point $x^-(v) \in [0; 1/2]$ and the point $x^+(v) \in [1/2; 1]$. The first point and its binary expansion have the form:

$$x^-(v) = 1/2 - F_{2v}(1/2)/4 = 0{,}x_1^-x_2^- \ldots,$$

where

$$x_n^- = (1 - c_{n-1})/2, \quad n = 1, 2, 3, \ldots.$$

The second point and its binary expansion have the form:

$$x^+(v) = 1 - x^-(v) = 1/2 + F_{2v}(1/2)/4 = 0{,}x_1^+x_2^+ \ldots,$$

where

$$x_n^+ = 1 - x_n^- = (1 + c_{n-1})/2, \quad n = 1, 2, 3, \ldots.$$

2) The global maximum of the function $T_v$ can be calculated using the formulas

$$T_v(x^{\pm}(v)) = \frac{1}{2(1-v)} - \frac{1}{4}\sum_{n=0}^{\infty} c_n \cdot (2v)^n \sum_{p=n}^{\infty} \frac{c_p}{2^p}$$

and

$$T_v(x^{\pm}(v)) = \frac{1}{2(1-v)} - \frac{1}{4\pi i}\int_{|z|=r} \frac{F_{2v}(z)F_{2v}(v/z)}{2z-1}dz,$$

where $r$ is any number from the interval $(\max(1/2, v), 1)$.

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# References

[1] O. Galkin, S. Galkina *Functions consistent with real numbers, and global extrema of functions in exponential Takagi class.* // arXiv:2003.08540v1 [math.CA], 2020.

[2] X. Han *On the Extrema of Functions in the Takagi Class.* // A thesis presented to the University of Waterloo in fulfillment of the thesis requirement for the degree of Master of Mathematics in Actuarial Science. Waterloo, Ontario, Canada, 2019.

[3] T. Takagi *A simple example of a continuous function without derivative.* // Tokyo Sugaku-Butsurigakkwai Hokoku, 1901. V. 1. P. 176–177.